

# ADMISSIBILITY OF ESTIMATORS OF THE PROBABILITY OF UNOBSERVED OUTCOMES\*

ARTHUR COHEN AND HAROLD B. SACKROWITZ

*Department of Statistics, Hill Center, Busch Campus, Rutgers University,  
New Brunswick, NJ 08903, U.S.A.*

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**Abstract.** The problem of estimating the probability of unobserved outcomes or, as it is sometimes called, the conditional probability of a new species, is studied. Good's estimator, which is essentially the same as Robbins' estimator, namely the number of singleton species observed divided by the sample size, is studied from a decision theory point of view. The results obtained are as follows: (1) When the total number of different species is assumed bounded by some known number, Good's and Robbins' estimators are inadmissible for squared error loss. (2) If the number of different species can be infinite, Good's and Robbins' estimators are admissible for squared error loss. (3) Whereas Robbins' estimator is a UMVUE for the *unconditional* probability of a new species obtained in one extra sample point, Robbins' estimator is not a uniformly minimum mean squared error unbiased estimator of the conditional probability of a new species. This answers a question raised by Robbins. (4) It is shown that for Robbins' model and squared error loss, there are admissible Bayes estimators which do not depend only on a minimal sufficient statistic. A discussion of interpretations and significance of the results is offered.

*Key words and phrases:* Probability of new species, Good's estimator, Robbins' estimator, admissibility, uniformly minimum variance unbiased, uniformly minimum mean squared error unbiased, sufficiency, completeness.