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PARAMETRIC STOCHASTIC CONVEXITY AND CONCAVITY OF STOCHASTIC PROCESSES*

MOSHE SHAKED[1] AND J. GEORGE SHANTHIKUMAR[2]

[1]*Department of Mathematics, University of Arizona, Tucson, AZ 85721, U.S.A.*
[2]*School of Business Administration, University of California, Berkeley, CA 94720, U.S.A.*

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Abstract. A collection of random variables $\{X(\theta), \theta \in \Theta\}$ is said to be parametrically stochastically increasing and convex (concave) in $\theta \in \Theta$ if $X(\theta)$ is stochastically increasing in θ, and if for any increasing convex (concave) function ϕ, $E\phi(X(\theta))$ is increasing and convex (concave) in $\theta \in \Theta$ whenever these expectations exist. In this paper a notion of directional convexity (concavity) is introduced and its stochastic analog is studied. Using the notion of stochastic directional convexity (concavity), a sufficient condition, on the transition matrix of a discrete time Markov process $\{X_n(\theta), n = 0, 1, 2, \dots\}$, which implies the stochastic monotonicity and convexity of $\{X_n(\theta), \theta \in \Theta\}$, for any n, is found. Through uniformization these kinds of results extend to the continuous time case. Some illustrative applications in queueing theory, reliability theory and branching processes are given.

Key words and phrases: Sample path convexity and concavity, Markov processes, directional convexity and concavity, single stage queues, supermodular and submodular functions, L-superadditive functions, reliability theory, branching processes, shock models, total positivity.