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BOOTSTRAPPING THE MODE

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Abstract. The problem of constructing bootstrap confidence intervals for the mode of a density is considered. Estimates of the mode are derived from kernel density estimates based on fixed and data-dependent bandwidths. The asymptotic validity of bootstrap techniques to estimate the sampling distribution of the estimates is investigated. In summary, the results are negative in the sense that a straightforward application of a naive bootstrap yields invalid inferences. In particular, the bootstrap fails if resampling is done from the kernel density estimate. On the other hand, if one resamples from a smoother kernel density estimate (which is necessarily different from the one which yields the original estimate of the mode), the bootstrap is consistent. The bootstrap also fails if resampling is done from the empirical distribution, unless the choice of bandwidth is suboptimal. Similar results hold when applying bootstrap techniques to other functionals of a density.

Key words and phrases: Bootstrap confidence intervals, mode, kernel density estimates.