

## OCCUPANCY WITH TWO TYPES OF BALLS

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**Abstract.** The classical occupancy problem is extended to the case where two types of balls are thrown. In particular, the probability that no urn contains both types of balls is studied. This is a birthday problem in two groups of boys and girls to consider the coincidence of a boy's and a girl's birthday. Let  $N_1$  and  $N_2$  denote the numbers of balls of each type thrown one by one when the first collision between the two types occurs in one of  $m$  urns. Then  $N_1N_2/m$  is asymptotically exponentially distributed as  $m$  tends to infinity.

This problem is related to the security evaluation of authentication procedures in electronic message communication.

*Key words and phrases:* Urn models, collisions, birthday problem,  $2 \times 2$  occupancy distribution, Stirling numbers of the second kind, compound binomial distribution, exponential distribution, Rayleigh distribution, cryptography.